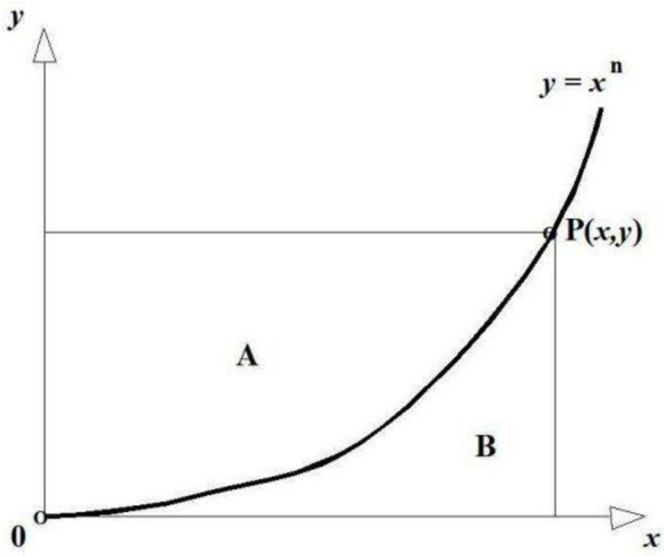
Go straight to the integral power rule

A pre-calculus taster by **Sidney Schuman** (Lewisham College 1980-1995)

First, a simple piece of algebra that produces an unexpected result.



Given that $\int x^n dx = \frac{x^{n+1}}{n+1}$ (constant of integration omitted)

then in the diagram, $B = \frac{x^{n+1}}{n+1}$ (Equation 1)

Also, $A + B = xy \quad \therefore A + B = x^{n+1} \quad \therefore A = x^{n+1} - B$

$\therefore A = x^{n+1} - \frac{x^{n+1}}{n+1} \quad \therefore A = n\frac{x^{n+1}}{n+1}$ (Equation 2)

Comparing equations 1 an 2, we see that $\frac{A}{B} = n$

Work for students

Based on the same diagram with, for example, $\boldsymbol{x = 10}$ and $\boldsymbol{n =}$ **2, 3, 4, 5 …** students can use the mid-ordinate rule with **10** vertical strips to calculate area **B** for each value of $\boldsymbol{n}$.

Area **A** can then be calculated for each value of **B** (since $A + B = xy$) to obtain the ratio $\frac{A}{B} = n$. (See Note 1)

Students can then be encouraged to logically deduce the integral power rule as follows:

$\frac{A}{B} = n \quad \therefore A = Bn \quad$ Now, $A + B = xy \quad \therefore Bn + B = xy \quad \therefore B(n+1) = xy \quad \therefore B = \frac{xy}{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$

Compare this result to the integral power rule as formally presented: $\int x^n dx = \frac{x^{n+1}}{n+1}$ (constant of integration omitted)

Note 1

Students will have realised that *exact* values of $\boldsymbol{n}$ are not obtained due to the small number of vertical strips used. Suggest that more accurate values could be obtained with a greater number of strips and ask them to consider how exact values could be obtained. This introduces the concept of infinitely large values (the number of vertical strips) and infinitely small values (the width of each vertical strip), helpful when they go on to learn calculus theory.

Note 2

Teaching calculus at Lewisham College, it was clear that some students were frightened by the word. Looking for an easier way to introduce it, I developed this method. Students worked with 'addition' rather than with the usual concept of gradient. Also a sense of achievement meant they were now more amenable to learning calculus theory.